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Exam II MTH 221, Fall 2016

Ayman Badawi

Total points $= \frac{?}{96}$

QUESTION 1. (7 points) Let $D = \{f(x) \in P_3 \mid f'(1) = 0 \text{ and } \int_0^1 f(x)\,dx = 0\}$. Convince me that D is a subspace of P_3. Find dim(D).

Idea: $D = \{ax^2+bx+c \mid 2a+b=0 \text{ and } \frac{1}{3}a+\frac{1}{2}b+c=0\}$

Now: solve system of L.E. 2X3

$$\begin{array}{ccc|c} a & b & c & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 1 & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 1 & 0 \end{array}\right] \xrightarrow{-\frac{1}{3}R_1+R_2\to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 1 & 0 \end{array}\right] \xrightarrow{3R_2} \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2+R_1\to R_1} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 1 & 3 & 0 \end{array}\right] \end{array}$$

$\Rightarrow a = \frac{3}{2}c,\ b = -3c,\ c \in \mathbb{R}$

$D = \{\frac{3}{2}cx^2 - 3cx + c \mid c \in \mathbb{R}\} =$

$D = \underbrace{\text{Span}\{\frac{3}{2}x^2 - 3x + 1\}}_{\text{subspace of } P_3}$. $\Longrightarrow \dim(D) = 1$

QUESTION 2. (5 points) Let $D = span\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}-1 & 2\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}\right\}$. Then D is a subspace of $R^{2\times 2}$. Find $dim(D)$ and Find a basis for D.

Normal question:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow[R_1+R_3\to R_3]{R_1+R_2\to R_2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow[-R_2+R_4\to R_4]{-2R_2+R_3\to R_3}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow \dim(D) = 2,$$ basis must be written as 2X2 matrix

$\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 1\\0 & 0\end{bmatrix}\right\}$.

QUESTION 3. (5 points) Give me a basis, call it B, for P_4 such that every polynomial in B is of degree 3.

Idea: P_4 is same as vector space as $\mathbb{R}^4$

so B must contain 4 independent polynomials.

so construct 4 rows in $\mathbb{R}^4$; 1st coordinate $\neq 0$

Independent

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix} \Rightarrow B = \{x^3, -x^3+x^2, -x^3-x^2+x, -x^3-x^2-x+1\}$$

QUESTION 4. (5 points) Given $A = \begin{bmatrix} 1 & a & b & 4 \\ 2 & 4 & c & 0 \\ 0 & d & -9 & 3 \\ 1 & -2 & 3 & 1 \end{bmatrix}$ is equivalent to the matrix $B = \begin{bmatrix} 1 & e & f & h \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Find the values of a, b, c, d.

Idea: Rank$(A) = 2$ (clear from B).

From B: 1st column and 4th column of A are indepen

Hence 2nd column of $A = (a, 4, d, -2) = \alpha_1(1,2,0,1) + \alpha_2(4,0,3,1) \Rightarrow 4 = 2\alpha_1 + 0 \Rightarrow \alpha_1 = 2$ and $-2 = \alpha_1 + \alpha_2 \Rightarrow -2 = 2 + \alpha_2 \Rightarrow \alpha_2 = -4$

$\Rightarrow a = 2 \times 1 + -4 \times 4 = -14$, $d = 2 \times 0 + -4 \times 3 = -12$

similarly: $(b, c, -9, 3) = \beta_1(1,2,0,1) + \beta_2(4,0,3,1)$

$-9 = 3\beta_2 \Rightarrow \beta_2 = -3$ and $3 = \beta_1 + \beta_2 \Rightarrow 3 = \beta_1 + -3$

$\Rightarrow \beta_1 = 6 \Rightarrow b = 6 \times 1 + -3 \times 4 = -6$, and $c = 6 \times 2 + -3 \times 0 = 6$

QUESTION 5. Given A is a 3×3 matrix such that 2 is an eiginvalue of A and $E_2 = span\{(1,2,-1),(0,-1,-4)\}$.

(i) **(5 points)** Can we conclude that $A \begin{bmatrix} 3 \\ 4 \\ -11 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -22 \end{bmatrix}$? EXPLAIN.

Idea: must show $(3,4,-11) \in span\{(1,2,-1),(0,-1,-4)\}$

$\Rightarrow (3,4,-1) = 3(1,2,-1) + 2(0,-1,-4) \Rightarrow$ Hence

$$A\begin{bmatrix} 3 \\ 4 \\ -11 \end{bmatrix} = 2\begin{bmatrix} 3 \\ 4 \\ -11 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -22 \end{bmatrix}$$

(ii) **(5 points)** If A is diagnolizable and $Trace(A) = 4$. Find $Rank(A)$. Is A invertible (nonsingular)? Explain.

Trace(A) = sum eigenvalues with multiplicaty

clearly 2 is repeated twice since $\dim(E_2) = 2$.

Hence trace $A = 2 + 2 + a = 4 \Rightarrow a = 0$. Since

3rd eigenvalue

0 eigenvalue $\Rightarrow |A| = 0 \Rightarrow A$ is not invertible

note $\dim(N(A)) = 1 \Rightarrow$ Rank$(A) = 2$.

QUESTION 6. let $A = \begin{bmatrix} 1 & 2 & -4 & 1 \\ -1 & -2 & 5 & -1 \\ 2 & 4 & -8 & 3 \end{bmatrix}$

(i) **(7 points)** Find $N(A)$ and then find a basis for $N(A)$

$Ax = 0 \quad Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -4 & 1 \\ -1 & -2 & 5 & -1 \\ 2 & 4 & -8 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -4 & 1 \\ -1 & -2 & 5 & -1 \\ 2 & 4 & -8 & 8 \end{bmatrix}$ $R_1+R_2 \to R_2$, $-2R_1+R_3 \to R_3$ $\begin{bmatrix} 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ $4R_2+R_1 \to R_1$ $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ $\frac{1}{6}R_3$ $\begin{array}{cccc} c_1 & & c_2 & c_3 \end{array}$ $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$-R_4+R_1 \to R_1$ $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$x_1 + 2x_2 + x_4 = 0$
$x_3 = 0$
$x_4 = 0$

$x_1 = -2x_2 - \cancel{x_4}$
x_2 free
$x_3 = 0$
$x_4 = 0$

soln $N(A) = \{(-2x_2, x_2, 0, 0) \mid x_2 \text{ free}\}$
$\{x_2(-2,1,0,0) \mid x_2 \text{ free}\}$
Span $\{(-2,1,0,0)\}$

Basis $\{(-2,1,0,0)\}$

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(ii) **(4 points)** Find a basis for $Col(A)$ (i.e., basis for the column space of A)

$Col(A)$ = span { independent columns (leading in A }
$Col(A)$ = span $\{(1,-1,2), (-4,5,-8), (1,-1,3)\}$
Basis = $\{(1,-1,2), (-4,5,-8), (1,-1,3)\}$

$\begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -8 \\ 1 & -1 & 3 \end{bmatrix}$ $4R_1+R_2 \to R_2$, $-R_1+R_3 \to R_3$

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ R_2+R_1 independent

4/4

(iii) **(3 points)** Does the point (5, 10, -19, 6) belong to Row(A)? EXPLAIN

Row (A) = span { independent rows in A }
Row(A) = span $\{(1,2,0,\cancel{0}), (0,0,1,0), (0,0,0,1)\}$

$a(1,2,0,\cancel{0}) + b(0,0,1,0) + c(0,0,0,1) = (5,10,-19,6)$

$a = 5 \quad 2a = 10 \quad b = -19$
$a = 5$

$c = 6$
$b = -19$
$a = 5$

Yes

$5(1,2,0,0) - 19(0,0,1,0) + 6(0,0,0,1) = (5,10,-19,0)$

the point $(5,10,-19,6)$ is a linear combination of any independent point in Row(A)

QUESTION 7. (5 points) Let $A = \begin{bmatrix} 1 & -2 & a & b \\ -1 & 4 & c & d \\ -1 & 2 & e & f \\ 1 & -2 & h & b+6 \end{bmatrix}$ where a, b, c, d, e, f, h are some numbers such that $|A| = 9$.

Consider the system of linear equations $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$. Find the value of x_3.

$$x_3 = \frac{\begin{vmatrix} 1 & -2 & 1 & b \\ -1 & 4 & 2 & d \\ -1 & 2 & 2 & f \\ 1 & -2 & 1 & b+6 \end{vmatrix}}{|A|} = \frac{36}{9} = 4$$

5/5

$\begin{vmatrix} 1 & -2 & 1 & b \\ -1 & 4 & 2 & d \\ -1 & 2 & 2 & f \\ 1 & -2 & 1 & b+6 \end{vmatrix}$ $R_1+R_2 \to R_2$, $R_1+R_3 \to R_3$, $-R_1+R_4 \to R_4$ $\begin{vmatrix} 1 & -2 & 1 & b \\ 0 & 2 & 3 & b+d \\ 0 & 0 & 3 & b+f \\ 0 & 0 & 0 & \cancel{b}+\cancel{b}+6 \end{vmatrix}$

$6\begin{vmatrix} 1 & -2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{vmatrix} = 6(1)(2)(3) = 36$

QUESTION 8. Let $A = \begin{bmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(i) (6 points) Find the eigenvalues of A.

$C_\alpha(A) = |\alpha I_3 - A| = 0$

$\left| \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} - \begin{bmatrix} 0 & 8 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right| = 0$

$\begin{vmatrix} \alpha & -8 & 0 \\ -1 & \alpha+2 & 0 \\ 0 & 0 & \alpha-3 \end{vmatrix} = 0$

$(\alpha-3)\begin{vmatrix} \alpha & -8 \\ -1 & \alpha+2 \end{vmatrix} = 0$

$(\alpha-3)[\alpha(\alpha+2) - 8] = 0$

$(\alpha-3)(\alpha-2)(\alpha+4) = 0$

$(\alpha-3)(\alpha^2 + 2\alpha - 8) = 0$

$\alpha_1 = 3 \quad \alpha_2 = 2 \quad \alpha_3 = -4$

✓ 6/6

(ii) (6 points) For each eigenvalue a of A find E_a.

$\alpha I_3 - A$ for $\alpha = 3$ $\quad E_3 = N(3I_3 - A)$ $\quad Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} \alpha & -8 & 0 \\ -1 & \alpha+2 & 0 \\ 0 & 0 & \alpha-3 \end{bmatrix}$ $\quad N\begin{bmatrix} 3 & -8 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6/6

$\begin{bmatrix} 3 & -8 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 & -8 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{3}R_1 \to R_1} \begin{bmatrix} 1 & -\frac{8}{3} & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 + R_2 \to R_2} \begin{bmatrix} 1 & -8/3 & 0 \\ 0 & 7/3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{3}{7}R_2 \to R_2} \begin{bmatrix} 1 & -8/3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\xrightarrow{\frac{8}{3}R_2 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad$ soln set $\{(0, 0, x_3) \mid x_3 \in \mathbb{R}\}$

$x_1 = 0$
$x_2 = 0$
x_3 free

$E_3 = \{(0, 0, x_3 \mid x_3 \in \mathbb{R}\}$

for $\alpha = 2$

$\begin{bmatrix} 2 & -8 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & -4 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_1 + R_2 \to R_2} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x_1 - 4x_2 = 0$
$x_1 = 4x_2$
x_2 free
$x_3 = 0$

$E_2 = \{(4x_2, x_2, 0) \mid x_2 \in \mathbb{R}\}$

$\alpha = -4$

$\begin{bmatrix} -4 & -8 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & -7 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_1} \begin{bmatrix} 1 & 2 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & -7 \end{bmatrix} \xrightarrow{R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -7 \end{bmatrix} \xrightarrow{-\frac{1}{7}R_3 \to R_3} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x_1 + 2x_2 = 0$
$x_2 = $ free
$x_3 = 0$

$E_{-4} = \{(-2x_2, x_2, 0) \mid x_2 \in \mathbb{R}\}$

(iii) (5 points) If A is diagnolizable, then find a diagonal matrix D and invertible matrix Q such that $A = QDQ^{-1}$.

$E_3 = \text{span}\{(0, 0, 1)\}$ $\quad$ $\dim E_3 = 1$

$E_2 = \text{span}\{(4, 1, 0)\}$ $\quad$ $\dim E_2 = 1$

$E_{-4} = \text{span}\{(-2, 1, 0)\}$ $\quad$ $\dim E_{-4} = 1$

$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

5/5

$Q = \begin{bmatrix} 0 & 4 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

 Lin Mahaini

QUESTION 11. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & -3 & 1 \end{bmatrix}$

(i) **(6 points)** Find the LU factorization of A (i.e., write $A = LU$ where L is invertible lower triangular and U is upper triangular).

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{3R_2+R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 3 & 1 \end{array}\right] \quad U \mid L^{-1}$$

$$I_3 \xrightarrow{-3R_2+R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \xrightarrow{-R_1+R_2 \to R_2} \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}}$$

6/6

$$A = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

(ii) **(3 points)** Use (i) to find the solution set to $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

Solution set $= \{(-3, 1, 5)\}$

$$\boxed{L^{-1}L}\,U \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = L^{-1} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

3/3

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$x_3 = 5 \qquad x_2 = 1 \qquad x_1 - x_2 + x_3 = 1, \quad x_1 = -3$

QUESTION 12. (3 points) Let A be a 3×3 matrix such that A is not diagnolizable and $C_A(\alpha) = (\alpha+5)^2(\alpha-3)$. Is it possible that A be invertible? if yes, then find $|A^{-1}|$. If no, then explain briefly.

Yes, $\quad |A| = (-5)^2 \times 3 = 75$

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{75}$$

3/3

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QUESTION 9. (5 points)

(a) (5 points) Use the least square method to find the best "fit" line of the form $y = ax + b$ to the points $(1,4), (-1,-4)$, and $(0,9)$.

x-value		y-value $y=ax+b$	given y-values
1	$\longrightarrow$	$a+b$	4
-1	$\longrightarrow$	$-a+b$	-4
0	$\longrightarrow$	b	9

$$\underbrace{\begin{array}{cc} a & b \\ \end{array}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 9 \end{bmatrix}$$

A

$$A^TA\begin{bmatrix} a \\ b \end{bmatrix} = A^T\begin{bmatrix} 4 \\ -4 \\ 9 \end{bmatrix} \Longrightarrow \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \Longrightarrow$$

$$\boxed{y = 4x + 3} \qquad a = 4 \Longrightarrow b = 3$$

(b) (2 points) What is the meaning that the line you found in (a) is the best "fit" to the given three points?

x-values	y-values $y=4x+3$	given y-value
1	7	4
-1	-1	-4
0	3	9

means $(4-7)^2 + (-4-(-1))^2 + (9-3)^2$ is minimum

$\leftarrow (\Delta y)^2 + (\Delta y)^2 + (\Delta y)^2$ is mininum

QUESTION 10. The following sets are not subspaces "vector spaces". For each give me an example to illustrate that one of the three axioms fails

(i) **(3 points)** $D = \{A \in R^{3\times3} \mid Rank(A) \le 2\}$ is not a subspace of $R^{3\times3}$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A, B \in D, \quad A + B = I_3 \notin D$$

(ii) **(3 points)** $D = \{f(x) \in P_3 \mid f(0) = 0 \text{ OR } f'(0) = 0\}$ is not a subspace of P_3.

$f_1(x) = x \in D$ since $f(0) = 0$
$f_2(x) = x - 1 \in D$ since $f(1) = 0$
$f_1 + f_2 = 2x - 1 \notin D$ since $(f_1+f_2)(0) \neq 0$ and $(f_1+f_2)(1) \neq 0$

(iii) **(3 points)** D = set of all points on the line $y = 4x + 1$ is not a subspace of R^2.

$(0,0) \notin D \Longrightarrow$ D is not subspace.

Exam II MTH 221, Fall 2016

Ayman Badawi

Total points $= \frac{?}{96} = 96$ Excellent +++

QUESTION 1. (7 points) Let $D = \{f(x) \in P_4 \mid f(1) = 0 \text{ and } f'(1) = 0\}$. Convince me that D is a subspace of P_3. Find dim(D).

2x

$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

$f(1) = a_3 + a_2 + a_1 + a_0 = 0 \to a_0 = -a_3 - a_2 - a_1 \to a_0 = -a_3 - a_2 - (-3a_3 - 2a_2)$

$a_0 = -a_3 - a_2 + 3a_3 + 2a_2$

$a_0 = a_2 + 2a_3$

$f'(x) = 3a_3x^2 + 2a_2x + a_1$

$f'(1) = 3a_3 + 2a_2 + a_1 = 0 \to a_1 = -3a_3 - 2a_2$

$\to D = \{a_3x^3 + a_2x^2 + (-3a_3 - 2a_2)x + (a_2 + 2a_3) \mid a_2, a_3 \in \mathbb{R}\}$

$\to D = \{a_3(x^3 - 3x + 2) + a_2(x^2 - 2x + 1) \mid a_2, a_3 \in \mathbb{R}\}$

$\to D = \text{span}\{x^3 - 3x + 2, x^2 - 2x + 1\}$ 4/4

Points: $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \end{bmatrix}$ ✓ independent

$\dim(D) = 2$

D is a subspace of P_4 because it can be written as a span. And span{any thing} is always a subspace of something. (Can't be a subspace of P_3 because it has a polynomial of degree 3 in the basis)

QUESTION 2. (5 points) Let $D = span\{x^2 + 1, -x^2 + x, -x^2 + 2x + 1, x + 1\}$. Then D is a subspace of P_3. Find $dim(D)$ and Find a basis for D.

$P_3 \longleftrightarrow \mathbb{R}^3$

$x^2 + 1$ $(1, 0, 1)$

$-x^2 + x$ $(-1, 1, 0)$

$-x^2 + 2x + 1$ $(-1, 2, 1)$

$x + 1$ $(0, 1, 1)$

$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $R_1 + R_2 \to R_2$, $R_1 + R_3 \to R_3$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ $-2R_2 + R_3 \to R_3$, $-R_2 + R_4 \to R_4$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \dim(D) = 2$

4/4

Basis: $\{x^2 + 1, -x^2 + x\}$

QUESTION 3. **(5 points)** Convince me that $D = \left\{ \begin{bmatrix} a+2b & -b \\ 3c & -c \end{bmatrix} \mid a, b, c \in R \right\}$ is a subspace of $R^{2\times2}$.

$$D = \left\{ a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 \\ 3 & -1 \end{bmatrix} \mid a, b, c \in R \right\}$$

$$\Rightarrow D = \text{span}\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3 & -1 \end{bmatrix} \right\}$$

Since D can be written as a span, it is a subspace of $R^{2\times2}$.

5/5

QUESTION 4. **(5 points)** Given $A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 4 & -1 \\ 2 & a & b & 9 \end{bmatrix}$ such that $Rank(A) = 2$. Find a, b.

Rank(A) = 2 $\longrightarrow$ 2 independent rows $\longrightarrow$ 3rd row is dependent (linear comb. of R_1 and R_2)

$k(1, 0, 2, 4) + m(0, 1, 4, -1) = (2, a, b, 9)$

$k = 2$

$4k - m = 9$

$8 - m = 9$

$m = -1$

$\Rightarrow 2(1, 0, 2, 4) - (0, 1, 4, -1) = (2, -1, 0, 9)$

$a = -1$

$b = 0$

5/5

QUESTION 5. Given A is a 3×3 matrix such that 2 is an eiginvalue of A and $E_2 = span\{(1, 2, -1), (0, -1, -4)\}$.

(i) **(5 points)** Can we conclude that $A \begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -12 \end{bmatrix}$? EXPLAIN. $A \begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix} = \underbrace{2}_{\text{Eigen value of } A} \begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix}$

5/5

$a(1, 2, -1) + b(0, -1, -4) = (2, 3, -6)$

$a = 2$

$2a - b = 3 \rightarrow 4 - b = 3 \rightarrow b = 1$

$-a - 4b = -6$ ✓

$-2 - 4 = -6$ ✓

Yes because the point $(2, 3, -6)$ belongs in E_2. It's a linear combinatio of $(1, 2, -1)$ and $(0, -1, -4)$. (the basis of E_2)

$2(1, 2, -1) + (0, -1, -4) = (2, 3, -6)$

(ii) **(5 points)** If A is diagnolizable and $Trace(A) = 10$. Find $|A|$.

A diagnolizable $\longrightarrow$ multiplicity of 2 is 2.

$2 + 2 + \alpha = 10$

$\alpha = 6 \rightarrow$ third eigen value is 6.

$|A| = 2 \times 2 \times 6 = 24$

5/5

QUESTION 6. let $A = \begin{bmatrix} 1 & 2 & -4 & 0 \\ -1 & -2 & 4 & 1 \\ 2 & 4 & -8 & 1 \end{bmatrix} \xrightarrow[-2R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \begin{bmatrix} 1 & 2 & -4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-R_2+R_3 \to R_3} \begin{bmatrix} 1 & 2 & -4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(i) **(7 points)** Find $N(A)$ and then find a basis for $N(A)$

$$\begin{bmatrix} 1 & 2 & -4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_4 = 0$

$x_1 + 2x_2 - 4x_3 = 0 \longrightarrow x_1 = -2x_2 + 4x_3$

$N(A) = \{(-2x_2+4x_3, x_2, x_3, 0) \mid x_2, x_3 \in \mathbb{R}\}$

$N(A) = \{x_2(-2,1,0,0) + x_3(4,0,1,0) \mid x_2, x_3 \in \mathbb{R}\}$

$N(A) = \text{span}\{(-2,1,0,0), (4,0,1,0)\}$ ✓

dim $N(A) = 2$

basis: $\{(-2,1,0,0), (4,0,1,0)\}$

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(ii) **(4 points)** Find a basis for $Col(A)$ (i.e., basis for the column space of A)

$\{(1,-1,2), (0,1,1)\}$ ✓

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(iii) **(3 points)** Does the point (4, -2, 10) belong to Col(A)? EXPLAIN

$a(1,-1,2) + b(0,1,1) = (4,-2,10)$

$a = 4$ ✓

$-a + b = -2$

$-4 + b = -2 \rightarrow b = 2$ ✓

$2a + b = 10$

$8 + 2 = 10$ ✓

Yes because it can be written as a linear combination of the basis of Col(A).

$4(1,-1,2) + 2(0,1,1) = (4,-2,10)$

3/3

QUESTION 7. **(5 points)** Let $A = \begin{bmatrix} 1 & -2 & 1 & a \\ -1 & 3 & b & c \\ -2 & 4 & 8 & d \\ -1 & 2 & -1 & e \end{bmatrix}$ where a, b, c, d, e are some numbers such that $|A| = 320$.

Consider the system of linear equations $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -8 \\ 6 \end{bmatrix}$. Find the value of x_4.

$$\begin{bmatrix} 1 & -2 & 1 & 2 \\ -1 & 3 & b & 6 \\ -2 & 4 & 8 & -8 \\ -1 & 2 & -1 & 6 \end{bmatrix} \xrightarrow[\substack{2R_1+R_3 \to R_3 \\ R_1+R_4 \to R_4}]{R_1+R_2 \to R_2} \begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & 1 & b+1 & 8 \\ 0 & 0 & 10 & -4 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

determinant: $10 \times 8 = 80$

$$x_4 = \frac{\begin{vmatrix} 1 & -2 & 1 & 2 \\ -1 & 3 & b & 6 \\ -2 & 4 & 8 & -8 \\ -1 & 2 & -1 & 6 \end{vmatrix}}{320}$$

$\rightarrow x_4 = \dfrac{80}{320} = \dfrac{1}{4}$ ✓

5/5

QUESTION 8. Let $A = \begin{bmatrix} 0 & 3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$AQ^T = \alpha Q^T \rightarrow AQ^T - \alpha Q^T = 0$

$(A - \alpha I_3) Q^T = 0$ $\quad |A - \alpha I_3| = 0$

$|\alpha I_3 - A| = 0$

(i) (6 points) Find the eigenvalues of A.

$$\alpha I_3 - A = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} - \begin{bmatrix} 0 & 3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & -3 & 0 \\ -1 & \alpha-2 & 0 \\ 0 & 0 & \alpha-1 \end{bmatrix}$$

$$|\alpha I_3 - A| \overset{\text{row } 3}{=} (\alpha-1)(-1)^6 \begin{vmatrix} \alpha & -3 \\ -1 & \alpha-2 \end{vmatrix} = \alpha-1\left(\alpha(\alpha-2)-3\right) = \alpha-1(\alpha^2-2\alpha-3)$$
$$=(\alpha-1)(\alpha-3)(\alpha+1)$$

$C_A(\alpha) = (\alpha-1)(\alpha-3)(\alpha+1)$

eigen values = 1, -1, 3 ✓

6/6

(ii) (6 points) For each eigenvalue a of A find E_a.

$$E_1 = N\begin{bmatrix} 1 & -3 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & -3 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_1+R_2 \to R_2}{\sim} \begin{bmatrix} 1 & -3 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 3x_2 = 0$
$x_2 = 0$
$x_1 = 0$
$x_3 \in \mathbb{R}$

$E_1 = \{(0, 0, x_3) \mid x_3 \in \mathbb{R}\} = \text{span}\{(0,0,1)\}$ ✓

$$E_{-1} = N\begin{bmatrix} -1 & -3 & 0 \\ -1 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad \begin{bmatrix} -1 & -3 & 0 \\ -1 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \overset{-R_1+R_2 \to R_2}{\sim} \begin{bmatrix} -1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$-x_1 - 3x_2 = 0$
$-x_1 = 3x_2$
$x_1 = -3x_2$
$x_3 = 0$

$E_2 = \{(-3x_2, x_2, 0) \mid x_2 \in \mathbb{R}\}$

6/6 $= \text{span}\{(-3, 1, 0)\}$ ✓

$$E_3 = N\begin{bmatrix} 3 & -3 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} 3 & -3 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \overset{\frac{1}{3}R_1}{\sim} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \overset{R_1+R_2 \to R_2}{\sim}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$x_1 - x_2 = 0 \rightarrow x_1 = x_2$
$x_3 = 0$

$E_3 = \{(x_2, x_2, 0) \mid x_2 \in \mathbb{R}\}$
$= \text{span}\{(1,1,0)\}$ ✓

(iii) (5 points) If A is diagnolizable, then find a diagonal matrix D and invertible matrix Q such that $A = QDQ^{-1}$.

Yes it is.

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} ✓ \quad Q = \begin{bmatrix} 0 & -3 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} ✓$$

5/5

QUESTION 9. (7 points) Use the least square method to find the best "fit" plane of the form $z = ax + by$ to the points $(1,1,1), (-1,1,-1)$, and $(0,2,4)$.

$a+b \quad 1$
$-a+b \quad -1$
$2b \quad 4$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$

$2a = 2$
$a = 1$
$6b = 8 \quad b = \frac{8}{6} = \frac{4}{3}$

best fit : $z = \frac{4}{3}y + x$

QUESTION 10. The following sets are not subspaces "vector spaces". For each give me an example to illustrate that one of the three axioms fails

3/3

(i) **(3 points)** $D = \{A \in R^{2\times 2} \mid |A| = 0\}$ is not a subspace of $R^{2\times 2}$.

addition (second axiom) fails

$f_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \in D$, $f_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \in D$

$f_1 + f_2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \notin D$

determinant $= 1$

(ii) **(3 points)** $D = \{ax^2 + x + b \mid a, b \in R\}$ is not a subspace of P_3.

addition (second axiom)

$f_1 = 2x^2+x+3 \in D$, $f_2 = 3x^2+x+1 \in D$

$f_1 + f_2 = 2x^2+x+3 + 3x^2+x+1$
$= 5x^2+2x+4 \notin D$

axiom fails

3/3

first and third axioms also fail.

(iii) **(3 points)** D = set of all points on the curve $y = 3x^2$ is not a subspace of R^2.

$(x, y) = (x, 3x^2)$

third axiom (scalar multiplication) fails.

$v = (1, 3) \in D$

$\alpha v \in D$? $\quad \alpha = -1 \rightarrow (-1, -3) \notin D$ axiom fails

3/3

QUESTION 11. Let $A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 2 & -1 & -1 \\ 0 & -3 & 1 & 6 \\ 1 & -1 & 1 & 0 \end{bmatrix}$

(i) **(6 points)** Find the LU factorization of A (i.e., write $A = LU$ where L is invertible lower triangular and U is upper triangular).

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 2 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 6 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-R_1+R_4 \to R_4]{R_1+R_2 \to R_2} \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 6 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{3R_2+R_3 \to R_3} \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$\underbrace{\qquad}_{U}$ $\underbrace{\qquad}_{L^{-1}}$

L: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-3R_2+R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_1+R_4 \to R_4]{-R_1+R_2 \to R_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}}_{L}$

$$\underbrace{\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 2 & -1 & -1 \\ 0 & -3 & 1 & 6 \\ 1 & -1 & 1 & 0 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U}$$

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(ii) **(2 points)** Find L^{-1}

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

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(iii) **(4 points)** Use (i) and (ii) to find the solution set to $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

$$L U \overbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}^{X} = \overbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}}^{C}$$

$LUX = C$

$L^{-1} L U X = L^{-1} C \to UX = L^{-1} C$

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ -2 \end{bmatrix}$$

$x_4 = -2$

$x_3 = 3$

$x_2 - 2x_4 = 1$

$x_2 + 4 = 1$

$x_2 = -3$

$x_1 - x_2 + x_3 - x_4 = 1 \to x_1 + 3 + 3 + 2 = 1 \quad x_1 = -7$

solution set $\{(-7, -3, 3, -2)\}$

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Faculty information

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